Business Education & Accreditation
Vol. 15, No. 1, 2023, pp. 51-67
ISSN: 1944-5903 (print)
ISSN: 2157-0809 (online)

www.theIBFR.COM

TIME VALUE OF MONEY TEACHING TOOL EXTENSIONS: THE INCLUSION OF GROWING ANNUITIES

Terrance Jalbert, University of Hawaii Hilo
Jonathan D. Stewart, Abilene Christian University

ABSTRACT

Earlier research develops tools to assist professors in teaching time value of money concepts. These tools systematically walk students through questions to identify the appropriate technique to solve a problem. This paper extends these tools by incorporating growing annuities into the analysis and giving users additional calculation capabilities. The paper develops a visual tool that provides users an easy method for identifying time value of money problems. The analysis shows calculations for growing annuities, including some previously unavailable calculations. The paper surveys textbooks to identify if and how growing annuities have been incorporated in business education pedagogy.

JEL: M40, M41, M52, A22, A23

KEYWORDS: Time Value of Money, Tools, Grown Annuities, Business Education

INTRODUCTION

Time value of money (TVM) concepts have proven difficult for teachers to communicate and for students to grasp. These difficulties are documented in the extant literature (Eddy and Swanson, 1996). Moreover, there exists variability in optimal pedagogy across students (Bloom, 1956). These issues motivate research to develop TVM teaching tools that accommodate the needs of various professors and students. Textbook presentation of TVM concepts commonly use inconsistent and misleading approaches as noted in Jalbert (2002). Text-book vagueness and impreciseness complicate user understanding of the materials. Efforts to clarify procedures and computations offer the possibility of improving student understanding and increasing mastery of the subject.

Earlier research provides tools to simplify the process of teaching TVM and provide students a systematic approach to solving problems. Jalbert (2002) and Jalbert, Jalbert and Chan (2004) first developed a visual technique selection tool which assists in solving TVM problems. This work was expanded upon by Martinez (2013), who includes calculator functions along with the visual presentation. Gardner (2004) suggests revisions to TVM calculations that include disregarding the beginning and ending terminology usually associated with teaching annuities. Jalbert (2002) provides tools to solve TVM problems including five questions users must answer which lead to six candidate techniques. Jalbert, Jalbert and Chan 2004 simplify the model by treating annuities as a special case of an uneven cash flow stream. Both tools systematically walk users through a series of simple and straightforward questions to identify correct techniques needed to solve TVM problems. This paper extends the work of Jalbert (2002) and Jalbert, Jalbert and Chan (2004) by expanding the tool to include growing annuity computations.

Growing annuities, sometimes called increasing annuities, provide useful computations for specific needs. This tool is especially useful for retirees who wish to assure a stream of annual payments that increase at the rate of inflation throughout their finite lifetime. A growing annuity involves a series of payments, limited in number, separated by an equal amount of time, with each successive payment growing at a constant percentage rate. To demonstrate the nature of a growing annuity, consider a series of annual cash flows. The first cash flow equals $5,000 received 1 year from today. The cash flows increase by 5 percent annually for the following 4 years, for a total of 5 payments. Timeline 1 depicts the cash flows.

```
0             1              2              3               4               5
|-------------|--------------|--------------|---------------|---------------|        (T1)

            5,000          5,250        5,512.50        5,788.125     6,077.531
```

This paper surveys introductory finance textbooks on their growing annuity calculation presentations. The survey also examines if texts provide a graphical TVM technique selection tool. The presentation provides a demonstration of growing annuity calculations including some new calculations not previously identified in the literature. The paper provides instructors a simple graphical method for presenting time value of money techniques in an easily understood format.

The remainder of the paper is organized as follows. In the next section we provide a review of the extant literature. The following section presents a graphical tool to frame growing annuities among other TVM techniques. The next section discusses the data and methodology used. The paper continues with a presentation of the results of our textbook survey. The next section presents growing annuity calculations. The paper closed with some concluding comments and suggestions.

LITERATURE REVIEW

There exists little literature related to the growing annuity technique. Thus, we relate the work here to the extant growing annuity literature as well as the general literature on time value of money techniques. The paper builds upon this research to develop a new graphical time value of money (TVM) technique selection tool. The paper also provides a comprehensive resource for growing annuity calculations.

Taylor (1986) demonstrated future value calculations for a growing ordinary annuity and growing annuity due. Hall (1996) discusses growing annuities as they relate to financial calculators. Others have provided instructions on how to complete the necessary computations using a financial calculator. Mayes (2023) demonstrated the necessary calculations using a Texas Instruments BAII Plus calculator. Omni Calculator (2023) provides an online tool to solve for future value, present values and payments of growing annuities. Michael's Law Firm (2023) provides the only known publicly available tool to solve for number of payments, the growth rate and the discount rate for growing annuities.

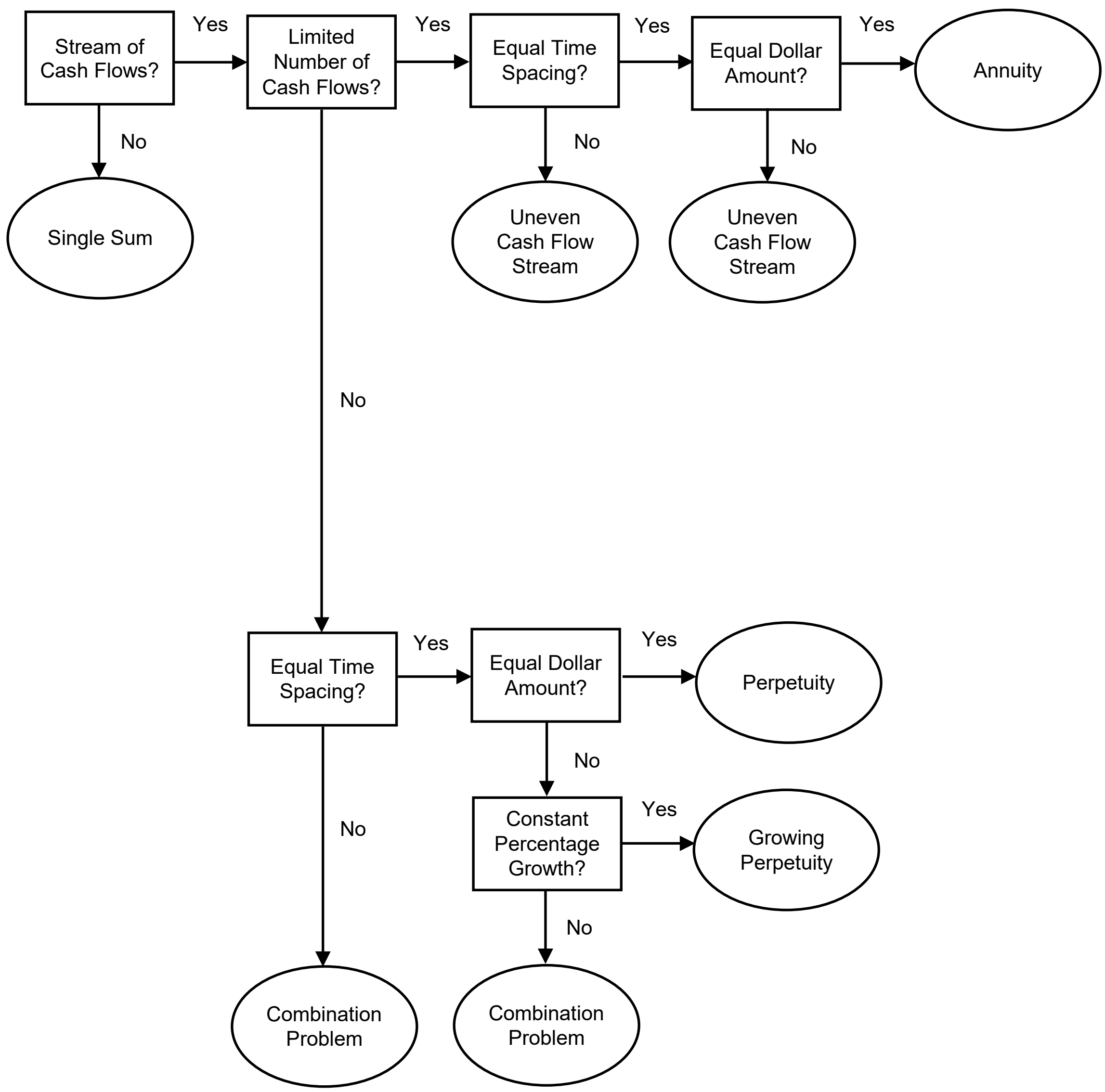
Bagamery (2011) provided a method for solving growing annuity problems that involves transforming a nominal stream of growing payments into a de-growthed stream of payments. He argued this approach makes the growing annuity approach more accessible to users. We extend the work of these authors by providing additional calculations related to growing annuities.

As noted earlier, Jalbert (2002) developed a teaching tool to assist in solving time value of money (TVM) problems as shown in Figure 1. The figure includes square boxes and ovals. Users evaluate five, yes or no, questions as indicted in boxes. The five questions are: 1. Is there a stream of cash flows? 2. Are the number of cash flows limited? 3. Is there an equal time spacing between each of the cash flows? 4. Is each cash flow for an equal dollar amount? and 5. Do the cash flows grow at a constant percentage rate? The question answers lead users to an oval which indicates the appropriate technique for solving a problem.

The tool includes six candidate techniques. Answering the questions correctly leads users unambiguously to the correct TVM technique.

To supplement the graph, Jalbert (2002), provided a summary of cash flow characteristics for each technique. Table 1, adapted from Jalbert (2002), shows the characteristics of each time value of money technique. A Yes indicator implies the technique requires the characteristic. A No indicator indicates the technique does not require the characteristic.

Figure 1: Time Value of Money Technique Selection Tool from Jalbert (2002)



This figure shows the time value of money technique selection tool developed in Jalbert (2002).

Table 1: Classification of Time Value of Money Problems

	Single Sum	Annuity	Perpetuity	Growing Perpetuity	Uneven Cash Flow Stream
Series of Cash Flows	No	Yes	Yes	Yes	Yes
Limited Number of Cash Flows	Yes	Yes	No	No	Yes
Each Cash Flow is for an Equal $ Amount	N/A	Yes	Yes	No	Yes/No
Equal Time Interval Between Cash Flows	N/A	Yes	Yes	Yes	Yes/No
Cash Flows Grow at a Constant Percentage Rate	N/A	No	No	Yes	No

This table shows time value of money technique selection criteria as adapted from Jalbert (2002).

Some students experience difficulty understanding and applying annuity techniques. As noted earlier, this stems in part from use of the terms beginning, end, ordinary annuity, and annuity due when describing annuity cash flows. It may also occur because individuals become overwhelmed with the number of techniques available. While annuity calculations provide added power, some individuals may not require the full range of capabilities that annuity techniques provide. These users can use a simplified method that treats annuities as a special cases of uneven cash flow streams. Jalbert, Jalbert and Chan (2004) provide a modification to the Jalbert (2002) technique that incorporates this reduced approach. Figure 2 and Table 2 show an adaptation of the revised technique. The reduced approach simplifies the TVM selection process by eliminating the annuity technique entirely. The reduced model results in one limitation. Financial calculators can solve for the payment amounts and number of observations in an annuity. However, they cannot do so when using the uneven cash flow stream tool. The simplified version proves valuable for those not needing these capabilities.

MODIFIED TIME VALUE OF MONEY TECHNIQUE SELECTION TOOL

We begin the analysis by providing a new graphical tool that walks users through the questions necessary to properly classify a time value of money problem. This paper extends the work of Jalbert 2002 and Jalbert, Jalbert and Chan (2004), by extending the graphical selection tool to include growing annuities.

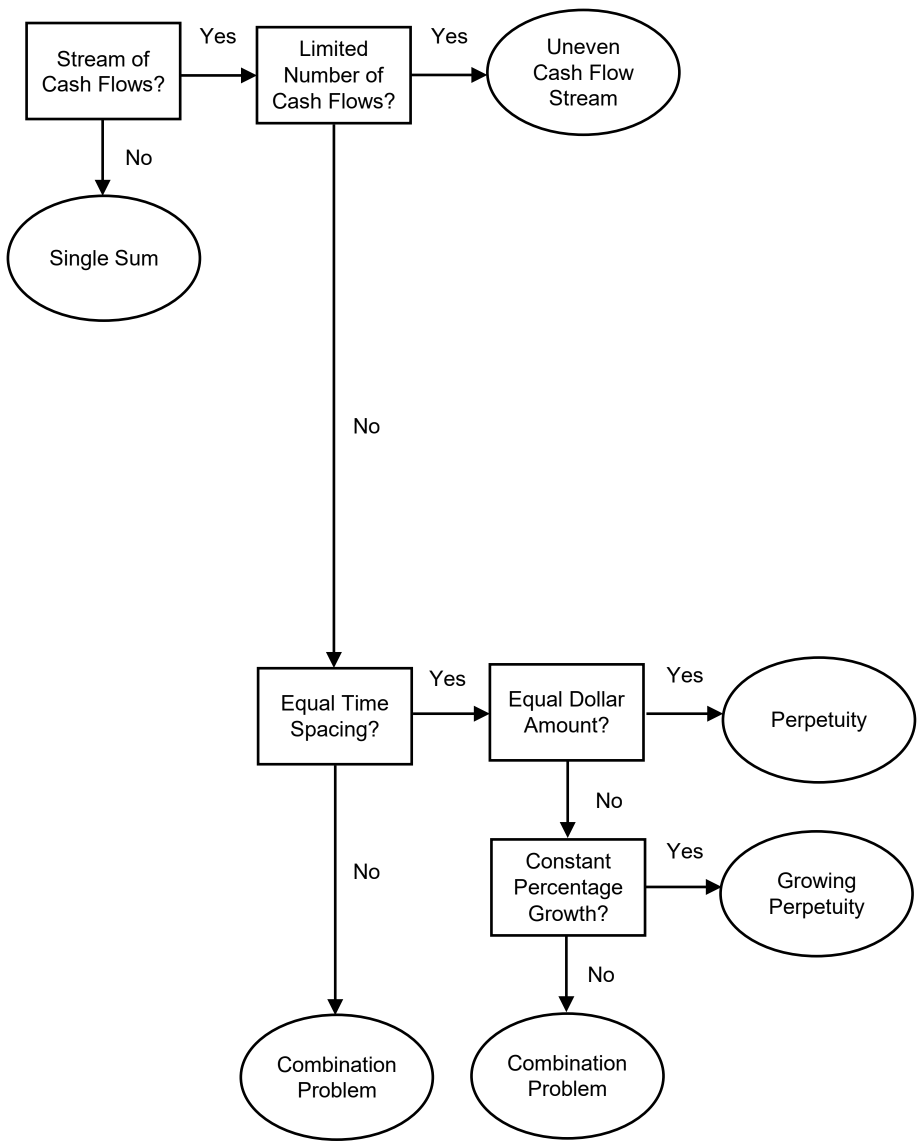
Specifically, the new tool modifies the Jalbert (2002) tool by incorporating the growing annuity technique. Figure 3 provides the tool. The new feature is incorporated under the Equal Dollar Amount question on the figure's top row. The figure shows a growing annuity involves a stream of cash flows, limited in number, with equal time spacing between each cash flow. The cash flows do not equal each other, however; the cash flows grow a constant percentage rate. Table 3 indicates characteristics of each available time value of money technique. Users reference Table 3 as an alternative to the graphical tool to select appropriate time value of money (TVM) techniques.

DATA AND METHODOLOGY

The analysis continues by evaluating finance textbooks to identify growing annuities coverage and the extent that textbooks include graphical time value of money (TVM) technique selection tools. The survey examines introductory and intermediate corporate finance textbooks along with personal finance textbooks. The sample of textbooks considered involves a convenience sample of fifteen texts. The sample includes thirteen corporate finance texts and two personal finance texts. Copyright on the texts range from 2000-2019.

The survey examines eight characteristics of the texts, seven related to growing annuity presentations and one related to inclusion of a graphical tool to assist users in selecting an appropriate TVM technique. The first survey item ascertains if the text addresses the basic growing ordinary annuity. For those texts that do present the growing annuity technique we examine a series of follow up issues. The first identifies if the text also addresses growing annuity due calculations. The next three elements assess if the presentation includes solving for present value, payment or future value of a growing annuity. The following item examines if the presentation demonstrates how to solve for other variables in the growing annuity, including number of periods, growth rate and discount rate. Next, we examine how the text addresses calculations. Three basic options are available including the formula method, spreadsheet method, and the calculator method. Finally, the survey examines if texts include a graphical selection tool, similar to that suggested in this research.

Figure 2: Time Value of Money Technique Selection Tool Excluding Annuities (Jalbert, Jalbert and Chan, 2004)



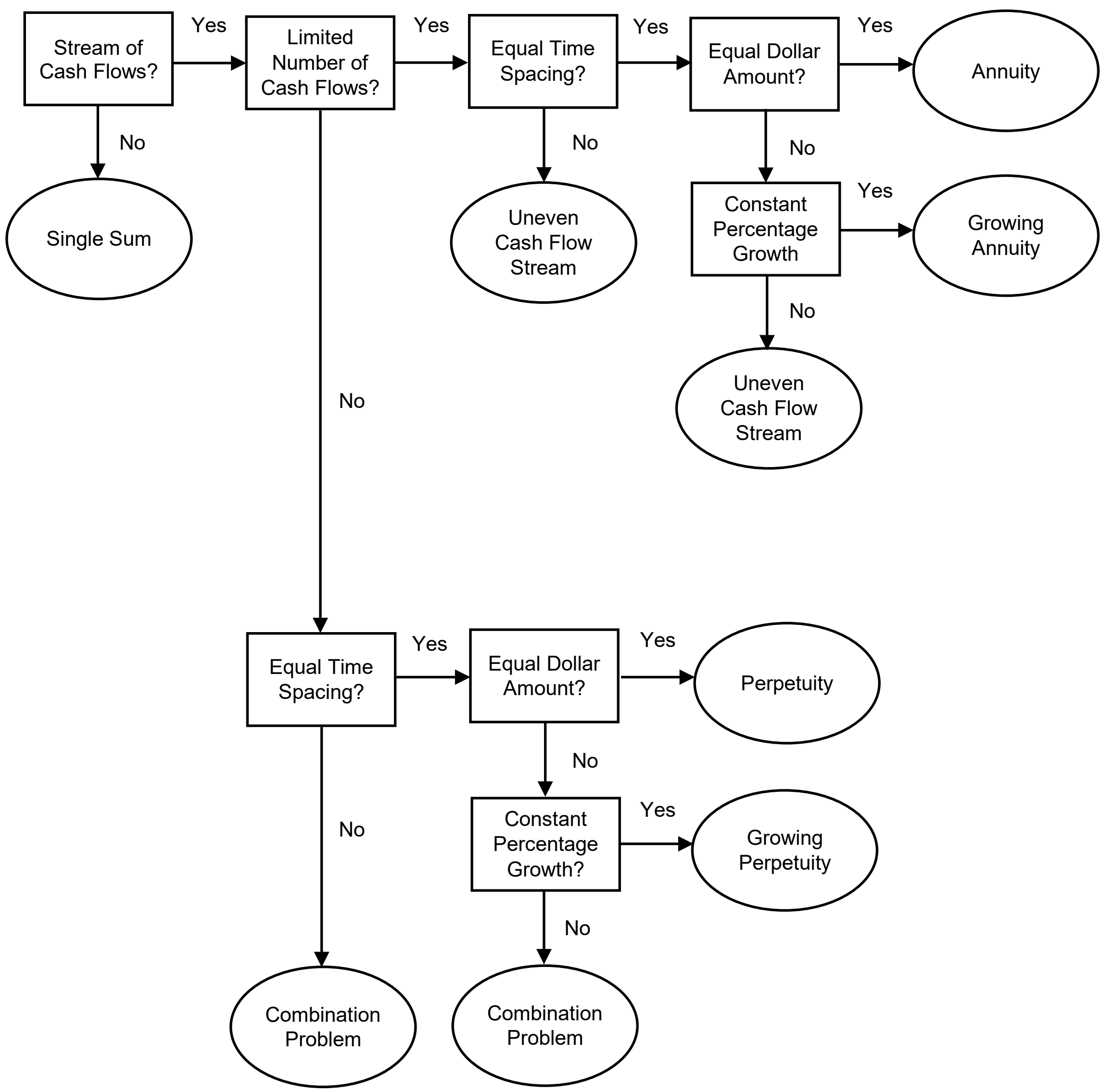
This figure shows the time value of money technique selection tool as adapted from Jalbert, Jalbert and Chan (2004).

Table 2: Modified Table for the Classification of Cash Flows

	Single Sum	Perpetuity	Growing Perpetuity	Uneven Cash Flow Stream
Series of Cash Flows	No	Yes	Yes	Yes
Limited Number of Cash Flows	Yes	No	No	Yes
Each Cash Flow is for an Equal $ Amount	N/A	Yes	No	Yes or No
Equal Time Interval Between Cash Flows	N/A	Yes	Yes	Yes or No
Cash Flows Grow at a Constant Percentage Rate	N/A	No	Yes	No

This table shows the time value of money selection criteria as adapted from Jalbert, Jalbert and Chan (2004).

Figure 3: Time Value of Money Technique Selection Tool Including Growing Annuities



This figure shows the time value of money technique selection tool developed here. The figure extends earlier works by incorporating the growing annuity technique.

Table 3: Classification of Time Value of Money Problems

	Single Sum	Annuity	Growin gAnnuit y	Uneven Cash Flow Stream	Perpetuity	Growing Perpetuity
Series of Cash Flows	No	Yes	Yes	Yes	Yes	Yes
Limited Number of Cash Flows	Yes	Yes	Yes	Yes	No	No
Each Cash Flow is for an Equal $ Amount	N/A	Yes	No	No	Yes	No
Equal Time Interval Between Cash Flows	N/A	Yes	Yes	No	Yes	Yes
Cash Flows Grow at a Constant Percentage Rate	N/A	No	Yes	No	No	Yes

This table shows the time value of money technique selection tool developed that includes the growing annuity technique.

RESULTS

Table 4 shows the survey results. Results reveal that growing annuities receive sparse attention in finance textbooks. Growing annuities are not commonly presented in textbooks. Only three of the thirteen corporate finance texts surveyed address the issue of growing annuities. The Ross, Westerfield, Jaffe and Jordan 2007, p. 115-116, Berk and DeMarzo 2014, p. 118-119 and Brigham and Ehrhardt (2017) texts do present growing annuity calculations. Neither of the personal finance texts present growing annuities.

Berk and Demarzo (2014) and Ross Westerfield, Jaffe and Jordan 2007 take similar approaches to growing annuities. Both texts demonstrate calculation of present and future values of a growing annuity. They do not address annuities due, solving for payment amounts or solving for other variables. Their presentation is done with formulas. Brigham and Ehrhardt (2017) provide arguably the most comprehensive coverage of growing annuities. Their text covers both ordinary growing annuities and growing annuities due. They demonstrate present value and payment calculations. They do not demonstrate how to solve for the future value or other variables. They use an indirect calculator method to demonstrate their calculations. None of the texts surveyed demonstrate calculation of number of periods, growth rate or discount rate. Overall, the survey shows that no text provides comprehensive coverage of the growing annuity technique. Despite the general lack of attention that growing annuities receive, they constitute an important tool in the time value of money (TVM) arsenal.

Examination of the texts for the presence of a graphical technique selection tool reveals that none of the surveyed texts incorporate a graphical selection tool. This is surprising given the added clarity that a graphical selection tool provides. This added clarity is particularly important for retirees who desire a stream of cash flows for a finite life that increase annually at the rate of inflation.

Table 4: Textbook Survey of Growing Annuity Coverage and Graphical Technique Selection Tool

Text	Growing Ordinary Annuities	Growing Annuities Due	Solve for Present Value	Solve for Payment	Solve for Future Value	Solve for other Variables	Calculation Method	Graphical Selection Tool
Panel A: Corporate Finance Texts								
Berk & Demarzo, 2014	Yes	No	Yes	No	Yes	No	Formula	No
Besley and Brigham, 2015	No	No	No	No	No	No		No
Bodie and Merton, 2000	No	No	No	No	No	No		No
Brigham and Daves, 2002	No	No	No	No	No	No		No
Brigham and Ehrhardt, 2017	Yes	Yes	Yes	Yes	No	No	Indirect Calculator	No
Brigham and Houston, 2019	No	No	No	No	No	No		No
Brooks, 2016	No	No	No	No	No	No		No
Foerster, S., 2015	No	No	No	No	No	No		No
Gitman, 2003	No	No	No	No	No	No		No
Keown, Martin and Petty, 2014	No	No	No	No	No	No		No
Moyer, McGuigan and Rao, 2015	No	No	No	No	No	No		No
Ross, Westerfield, Jaffe and Jordan, 2007	Yes	No	Yes	Yes	No	No	Formula	No
Smart, Megginson and Gitman, 2007	No	No	No	No	No	No		No
Panel B: Personal Finance Texts								
Gitman, Joehnk and Billingsley, 2016	No	No	No	No	No	No		No
Madura, 2006	No	No	No	No	No	No		No

This table provides an examination of finance textbook coverage of growing annuities. It also shows the extent to which texts provide a graphical selection tool for identifying appropriate time value of money techniques. Texts selected for inclusion represent a convenience sample.

GROWING ANNUITIES

This section shows growing annuity calculations. The discussion adds to existing literature by aggregating various previously presented calculations into a single location for easy access. It also provides relevant calculations not previously presented in textbooks or the extant literature.

Recall that a growing annuity has the following characteristics: 1. There is a series of cash flows, 2. The number of cash flows is limited, 3. There is an equal time spacing between each cash flow, 4. Each cash flow is not for an equal amount, and 5. The cash flows become larger by some constant percentage amount in each subsequent year. Growing annuities differ from standard annuities. A growing annuity includes periodic increasing payments. In contrast, a standard annuity is characterized by equal periodic payments. Growing annuities also differ from growing perpetuities. Both a growing perpetuity and a growing annuity have periodic increasing payments. However, growing perpetuity payments continue into infinity while growing annuity payments have a defined end point.

We return to the growing annuity example presented earlier. Recall the growing annuity example involved a series of annual cash flows. The first cash flow equals $5,000, received 1 year from today. Cash flows increase by five percent in each year for the following four years, for a total of five payments. Timeline T1 depicts the cash flows.

```
 0             1              2              3               4               5
 |-------------|--------------|--------------|---------------|---------------|        (T1)

               5,000          5,250          5,512.50        5,788.125       6,077.531
```

<u>Future Value Based Calculations</u>

We examine growing annuities by calculating the usual time value of money (TVM) parameters. We begin with formula for the future value (FV) of the growing annuity. Consider a growing annuity where PMT_1 equals the payment received one year from today, PMT_0 equals the payment received one minute from now, N equals the number of years payments will occur, I equals the interest earned on investments and G equals the growth rate in investments. *FVGOA* indicates the future value of a growing ordinary annuity and *FVGAD* indicates the future value of a growing annuity due. Equations 1 and 2 show calculations for *FVGOA* and *FVGAD* respectively:

$$FVGOA = PMT_1(\frac{(1+I)^N-(1+G)^N}{I-G}) \tag{1}$$

$$FVGAD = PMT_0(1+I)(\frac{(1+I)^N-(1+G)^N}{I-G}) \tag{2}$$

To demonstrate the calculation of Equation 1, consider the growing annuity noted above. Further note the interest rate earned on investments equals 7 percent. Then the future value of the growing annuity equals:

$$FVGOA = 5{,}000(\frac{(1+0.07)^5-(1+0.05)^5}{0.07-0.05})$$

$$FVGOA = 5{,}000(\frac{1.402551731-1.276281562}{0.02}) = 31{,}567.54213$$

Thus, an investor making the payments noted in the problem will accumulate \$31,567.54213 at the end of the 5th year.

Now suppose, in the previous problem, that the first payment occurs at time zero rather than at time 1. The total number of cash flows remains at five and we wish to know the value of the stream at the end of the 5th year. Timeline T2 depicts the cash flows as follows:

```
 0             1              2              3               4               5
 |-------------|--------------|--------------|---------------|---------------|        (T2)

5,000          5,250          5,512.50       5,788.125       6,077.531
```

In this case the *FVGAD* approach, given by Equation 2, applies with calculations equaling:

$$FVGAD = 5{,}000(1+0.07)(\frac{(1+0.07)^5-(1+0.05)^5}{0.07-0.05}) \tag{2}$$

$$FVGAD = 5{,}000(1+0.07)\left(\frac{1.402551731-1.276281562}{0.02}\right) = 33{,}777.27021$$

Thus, an investor who makes the deposits as noted will accumulate \$33,777.27021 at the end of the 5th year.

Rearranging the formulas solves for the initial payment when knowing the future value, rate earned on investments, growth and number of periods as follows for an ordinary growing annuity and growing annuity due respectively:

$$PMT_1 = \frac{FVGOA}{(\frac{(1+I)^N-(1+G)^N}{I-G})} \tag{3}$$

$$PMT_0 = \frac{FVGAD}{(1+I)(\frac{(1+I)^N-(1+G)^N}{I-G})} \tag{4}$$

To demonstrate the use of Equation 3, consider an investor who wishes to accumulate 31,567.54213, five years from today. The investor makes five annual payments into the account to achieve the goal with the first payment occurring one year from today and the last payment occurring five years from today. Payments increase by five percent per year. The account pays seven percent interest annually. The investor desires to know the first payment amount necessary to achieve the goal. The following equation solves for the initial payment amount:

$$PMT_1 = \frac{31{,}567.54213}{(\frac{(1+0.07)^5-(1+0.05)^5}{0.07-0.05})}$$

$$PMT_1 = \frac{31{,}567.54213}{(\frac{1.402551731-1.276281562}{0.02})} = \$5{,}000$$

Similarly, if the payments occur at the start of each year, as in the case of a growing annuity due, we solve for equation 4 as follows:

$$PMT_0 = \frac{33{,}777.27021}{(1+0.07)(\frac{(1+0.07)^5-(1+0.05)^5}{0.07-0.05})}$$

$$PMT_0 = \frac{33{,}777.27021}{(1+0.07)(\frac{1.402551731-1.276281562}{0.02})} = 5{,}000$$

<u>Present Value Based Calculations</u>

Next, we calculate present values. Equations 5 and 6 show formulas for calculating the present value of a growing ordinary annuity and a growing annuity due respectively:

$$PVGOA = \frac{PMT1}{I-G}[1-\left(\frac{1+G}{1+I}\right)^N] \tag{5}$$

$$PVGAD = \frac{PMT0(1+G)}{I-G}\left[1-\left(\frac{1+G}{1+I}\right)^{N-1}\right] + PMT0 \tag{6}$$

Consider an investor who wishes to withdraw \$5,000 from an account at the end of year one. The investor continues to withdraw funds from the account annually for each of the next four years, for a total of five payments. Payments increase by five percent annually and the account pays seven percent interest annually. The investor wishes to know the deposit required today to achieve this objective. Equation 5 calculates the present value of the growing ordinary annuity as follows:

$$PVGOA = \frac{5{,}000}{.07-.05}\left[1-\left(\frac{1+.05}{1+.07}\right)^{5}\right] = 22{,}507.2212$$

The result indicates that achieving a five-year growing annuity as specified requires an initial deposit of \$22,507.2212.

Next, consider an investor who wishes to withdraw \$5,000 from an account at the beginning of year one. The investor continues to withdraw funds from the account annually each of the next four years, for a total of five payments. Payments increase by 5 percent annually and the account pays seven percent interest annually. The investor wishes to know the deposit required today to achieve this objective. Equation 6 calculates the present value of the growing ordinary annuity as follows:

$$PVGAD = \frac{5{,}000(1+0.05)}{0.07-0.05}\left[1-\left(\frac{1+0.05}{1+0.07}\right)^{5-1}\right] + 5{,}000 = 24{,}082.72669$$

The result indicates that achieving the five-year growing annuity, as specified, requires an initial deposit of \$24,082.72669.

The presentation continues by providing alternate formulae for calculating payment amounts in a growing annuity. Equations 7 and 8 provide formulae for calculating the initial growing annuity payment with a known present value. Rearranging equations 5 and 6 solves for the payment amount as shown in Equations 7 and 8 for a growing ordinary annuity and growing annuity due respectively.

$$PMT_1 = \frac{PVGOA}{[1-\left(\frac{1+G}{1+I}\right)^{N}]}\,(I-G) \tag{7}$$

$$PMT_0 = \frac{PVGAD}{\frac{1+G}{I-G}\left[1-\left(\frac{1+G}{1+I}\right)^{N-1}\right]+1} \tag{8}$$

To demonstrate use of Equation 7, consider an investor with $22{,}507.2212$ in an account that pays seven percent interest annually. The investor withdraws annual payments from the account at the end of the following five years to empty the account. The payments grow at a rate of five percent annually. How much can the investor withdraw from the account at the end of the first year? Equation 7 calculates the time zero payment as follows:

$$PMT_1 = \frac{22{,}507.2212}{[1-\left(\frac{1+0.05}{1+0.07}\right)^{5}]}\,(0.07-0.05) = 5{,}000$$

To demonstrate the use of Equation 8, consider an investor with \$24,082.72669 in an account that pays 7 percent interest per year. The investor withdraws annual payments from the account at the beginning of the following 5 years to empty the account. The payments grow at a rate of 5 percent annually. How much can the investor withdraw from the account at the beginning of the first year? Equation 8 calculates the payment at time zero as follows:

$$PMT_0 = \frac{24{,}082.72669}{\frac{1+0.05}{0.07-0.05}\left[1-\left(\frac{1+0.05}{1+0.07}\right)^{4}\right]+1} = 5{,}000$$

We continue with formulas to solve for *N* in a growing annuity with a known present value. To the best of our knowledge, this presentation represents the first discussion of formula for completing the task.

Equations 9 and 10 show the formulas for computing *N* in an ordinary growing annuity and a growing annuity due respectively.

$$N = ln - \left[\frac{PVGOA}{\left(\frac{PMT_1}{I-G}\right)} - 1\right] * \left[\frac{1}{ln\left(\frac{1+G}{1+I}\right)}\right] \tag{9}$$

$$N = \left\{ln - \left[\left(\frac{(PVGAD - PMT_0)*(I-G)}{PMT_0(1+G)}\right) - 1\right] * \left[\frac{1}{ln\left(\frac{1+G}{1+I}\right)}\right]\right\} + 1 \tag{10}$$

To demonstrate the use of Equation 9, consider an investor who wishes to accumulate 31,567.54213. The investor will make annual payments into the account to achieve the goal with the first payment of $5,000 occurring one year from today and the last payment occurring when the goal is achieved. The payments increase by five percent annually and the account pays seven percent interest annually. The investor desires to know how long it will take to achieve the goal. The following application of Equation 9 computes the result:

$$N = ln - \left[\frac{22{,}507.2212}{\left(\frac{5{,}000}{0.07-0.05}\right)} - 1\right] * \left[\frac{1}{ln\left(\frac{1+0.05}{1+0.07}\right)}\right] = 5$$

Thus, under the scenario presented, the investor accumulates $31,567.54213 at the end of the 5th year.

To demonstrate the use of Equation 10, consider an investor desiring to accumulate 31,567.54213. The investor makes annual payments into an account to achieve the goal with the first payment of $5,000 occurring later today and the last payment occurring when the goal is achieved. The payments increase by five percent annually and the account pays seven percent interest annually. The investor desires to know how long it will take to achieve the goal. The following application of Equation 10 shows the result:

$$N = \left\{ln - \left[\left(\frac{(24{,}082.72669 - 5{,}000)*(0.07-0.05)}{5{,}000(1+0.05)}\right) - 1\right] * \left[\frac{1}{ln\left(\frac{1+0.05}{1+0.07}\right)}\right]\right\} + 1 = 5$$

The result shows, under the scenario presented, the investor accumulates$24,082.72669 at the 5th year end.

<u>Solving for other variables</u>

On occasion it is useful to calculate other amounts. Given changing payment amounts, users might desire to know the payment at various points of time in the growing annuity. Equations 11 and 12 calculate the payment amount at any time point for a growing ordinary annuity and growing annuity due respectively.

$$PMT_t = PMT_1(1+G)^{N-1} \tag{11}$$

$$PMT_t = PMT_0(1+G)^{N} \tag{12}$$

The first known introduction of Equation 11 was done by Finance Formulas.Net (2023). This is the first known presentation of Equations 12. We demonstrate the calculations using the ordinary growing annuity example above for the payment at time four, with an initial payment of $5,000 that grows at a rate of 5 percent annually. The computations show a future value of $5,788.125 as follows:

$$PMT_4 = 5{,}000(1+0.05)^{4-1} = 5{,}788.125$$

To demonstrate the use of equation 12 we use the growing annuity due example noted above, with a Time 0 payment of $5,000 and a growth rate of 5 percent annually, the payment at time 4 equals:

$$PMT_4 = 5{,}000(1 + 0.05)^4 = \$6{,}077.531$$

It is possible to solve the equations for I, G. However, no direct formula is available for solving for these variables. Rather, solving for these variables is an iterative process. As such, users must vary values for the variable of interest, until the correct solution is achieved. As noted earlier, Michael's Law Firm (2023) provides the only known publicly available tool to solve for number of payments, the growth rate and the discount rate for growing annuities.

For brevity, we limit ourselves to the above calculations. However, the extant literature provides additional calculation tools for working with growing annuities that may be valuable for some users. Carbon Collective (2023) provides a formula for calculating the FV of a growing annuity when I = G as:

$$FVGOA = PMT_1N(1 + r)^{N-1}$$

Taylor (1986) provides formulas for situations where compounding occurs more than once per year. In addition, Taylor (1986) provides formulas for a situation where payments occur more than once per year but increase annually.

<u>Growing Annuity Value Table</u>

Growing annual balances display interesting patterns with important implications for investors. To demonstrate these patterns, we examine a longer-term growing annuity. Consider a growing ordinary annuity, with an initial balance of $100,000. The first annual withdrawal of $4,627.070927 occurs at the end of the first year and the payments continue for thirty years. The annual growth rate in withdrawals equals five percent. The account pays a return of seven percent annually.

Table 4 shows the pattern of withdrawals and account balances throughout the thirty-year period. The first column indicates the year from 1 through the 30-year life of the growing annuity. The Beg. Balance column indicates beginning of the year funds held in the account. The column labeled Earnings indicates the amount of interest earned on the account during the year. The column labeled Withdrawal indicates the amount of money removed from the account at year end. Finally, the column labeled End Balance indicates the amount of money remaining in the account at year end.

Table 4: Growing Annuity Value Table

Year	Beg Balance	Earnings	Withdrawal	End Balance
1	100,000.00	7,000.00	4,627.07	102,372.93
2	102,372.93	7,166.11	4,858.42	104,680.61
3	104,680.61	7,327.64	5,101.35	106,906.91
4	106,906.91	7,483.48	5,356.41	109,033.98
5	109,033.98	7,632.38	5,624.23	111,042.12
6	111,042.12	7,772.95	5,905.45	112,909.63
7	112,909.63	7,903.67	6,200.72	114,612.58
8	114,612.58	8,022.88	6,510.75	116,124.71
9	116,124.71	8,128.73	6,836.29	117,417.15
10	117,417.15	8,219.20	7,178.11	118,458.24
11	118,458.24	8,292.08	7,537.01	119,213.31
12	119,213.31	8,344.93	7,913.86	119,644.38
13	119,644.38	8,375.11	8,309.55	119,709.93
14	119,709.93	8,379.70	8,725.03	119,364.59
15	119,364.59	8,355.52	9,161.28	118,558.83
16	118,558.83	8,299.12	9,619.35	117,238.60
17	117,238.60	8,206.70	10,100.32	115,344.99
18	115,344.99	8,074.15	10,605.33	112,813.80
19	112,813.80	7,896.97	11,135.60	109,575.17
20	109,575.17	7,670.26	11,692.38	105,553.06
21	105,553.06	7,388.71	12,277.00	100,664.77
22	100,664.77	7,046.53	12,890.85	94,820.46
23	94,820.46	6,637.43	13,535.39	87,922.50
24	87,922.50	6,154.58	14,212.16	79,864.92
25	79,864.92	5,590.54	14,922.77	70,532.70
26	70,532.70	4,937.29	15,668.90	59,801.08
27	59,801.08	4,186.08	16,452.35	47,534.81
28	47,534.81	3,327.44	17,274.97	33,587.28
29	33,587.28	2,351.11	18,138.72	17,799.67
30	17,799.67	1,245.98	19,045.65	0.00

This table shows the payoff pattern of a growing ordinary annuity. The growing ordinary annuity has an initial balance of $100,000 and an end of Year 1 withdrawal of $4,627.070927. Payments continue for thirty years with an annual payment growth rate equaling five percent. The account pays a return of seven percent per year.

A pattern stands out in Table 4. The careful reader will notice the End Balance increases during the first thirteen years of the thirty-year growing annuity. After year thirteen the balance declines until it reaches a balance of zero at the end of year thirty. This pattern is especially important for retirees planning their retirement spending. Retirees face the temptation to increase their spending in light of increasing account balances in years 1-13. It is important for retirees to recognize this account balance increase offsets planned spending in subsequent years and does not imply a higher initial spending level.

CONCLUDING COMMENTS

Growing annuity techniques provide important insights and capabilities to financial planners. This holds especially true for retirees who wish to create a stream of cash flows that increase with inflation throughout their finite life. This paper provides a comprehensive summary of growing annuity calculation methods. It also provides a graphical tool to assist users in identifying the appropriate technique to apply to any time value of money (TVM) problem. The tools presented here provide instructors an intuitive way to present TVM techniques that students can easily grasp and master.

We survey a convenience sample of introductory finance textbooks. Results show that most texts do not incorporate growing annuities into their presentation. Moreover, texts that do include growing annuities provide a limited discussion. Nevertheless, growing annuities represent an important TVM tool. We encourage textbook authors to fully incorporate the growing annuity technique as presented here.

The survey further reveals that textbooks do not present a graphical TVM technique selection tool. These tools provide a simple way for users to identify the appropriate approach to solve a problem. We encourage textbook authors to incorporate graphical tools into their TVM presentations to facilitate better, and easier student understanding.

We note that no known financial calculator incorporates growing annuities into their tool set. This holds despite incorporating similar functions such as annuities and perpetuities. The presence of these related tools suggest incorporating growing annuities would be a manageable task. We encourage financial calculator manufacturers to incorporate these tools into calculator functions.

REFERENCES

Bagamery (2011) "A Calculator-Friendly Transformation Method for Valuing Finite Growing Annuities and Annuities Due, *Journal of Financial Education,* Vol. 37(1/2, Spring\Summer) p. 83-100

Berk J. and P. DeMarzo (2014), *Corporate Finance,* 3rd Ed. Pearson, Boston p. 118-121

Besley and Brigham (2015), *CFIN4,* 1st Edition, Cengage Learning, Stamford, CT, p. 57-72

Bloom, B. (1956). *Taxonomy of Educational Objectives, Handbook I: Cognitive Domain*, New York: McKay.

Bodie, Z., and R. C. Merton, (2000) *Finance*, 1st Ed. Upper Saddle River, New Jersey, Prentice-Hall

Brigham E.F and P.R. Daves (2002), *Intermediate Financial Management,* 7th Ed., South-Western, Thompson Learning, p. 370-415

Brigham, E. and M. Ehrhardt, *Financial Management Theory and Practice*, 15th Ed., Boston, Cengage Learning, 2017, p. 179-181

Brigham, E. F. and J. F. Houston, (2019), *Fundamentals of Financial Management*, 15th Ed., Cengage Learning, Boston, MA, p. 148-191

Brooks, R. M. (2016), *Financial Management Core Concepts,* 3rd Ed., Pearson Education, Inc., Upper Saddle River New Jersey, p. 54-143

Carbon Collective (2021) "Future Value of a Growing Annuity," *Carbon Collective*, March 24, 3021. Accessed February 9, 2023 from: https://www.carboncollective.co/sustainable-investing/future-value-of-a-growing-annuity

Eddy, Albert and Gene Swanson (1996), "A Hierarchy of Skills Approach to Teaching Accounting Present Value," *Journal of Accounting Education 14(1) p. 123-131*

Finance Formulas (2023) "Growing Annuity Payment – PV," Accessed on February 10, 2023 from: https://www.financeformulas.net/Growing-Annuity-Payment.html#:~:text=The%20formula%20for%20calculating%20the%20initial%20payment%20on,shown%20directly%20above%2C%20which%20can%20be%20shown%20as

Foerster, S. (2015) *Financial Management Concepts and Applications,* 1st Ed., Pearson Education, Inc., Upper Saddle River New Jersey, p. 129-166

Gardner, N.D. (2004) "The Time Value of Money: A Clarifying and Simplifying Approach," *Journal of College Teaching & Learning,* Vol. 1(7), p. 25-29

Gitman, L.J. (2003) *Principles of Managerial Finance*, 10th Ed., Pearson Education, Inc. Boston, MA, p. 148-211

Gitman, L.J., M.D. Joehnk and R.S. Billingsley, (2016), *PFIN 4,* 1st Ed., Cengage, Boston, MA, p. 45-50

Hall, P.L. (1996) "Growing Annuities and the Financial Calculator," *Journal of Financial Education,* Vol 22, pl 73-75

Jalbert, Terrance (2002) "A New Method for Teaching the Time Value of Money," Terrance Jalbert, *Journal of the American Academy of Business, Cambridge* Vol. 2(1), September 2002 p. 72-79

Jalbert, Terrance, Mercedes Jalbert and Wai Yee Canri Chan (2004) "Advances in Teaching the Time Value of Money," *Journal of College Teaching and Learning*, Vol. 1(8), August, p. 7-12

Keown, A., J.D. Martin, and J. W. Petty, *Foundations of Finance: The Logic and Practice of Financial Management,* 8th Ed., Upper Saddle River, New Jersey, Pearson Education Inc, 2014

Madura, J. (2006), *Personal Finance, 2nd Ed.,* Pearson, Addison Wesley, Boston, MA, p. 59-84

Martinez, Valeria, (2013) "Time Value of Money Made Simple: A Graphic Teaching Method," *Journal of Financial Education,* Vol. 39 N 1/2, Spring/Summer, p. 96-117

Mayes, T. R. (2023) "Graduated Annuities on the BAII Plus," *TVMCalcs.com* accessed February 6, 2023 at: http://www.tvmcalcs.com/index.php/calculators/apps/ti-baii-plus-graduated-annuities

Michael's Law Firm (2023) "Future Value of Growing Annuity Calculators – Ordinary Growing Annuity and Growing Annuity Due," Viewed, March 17, 2023 at: https://www.michaelsfirm.ca/future-value-of-growing-annuity-calculators-ordinary-growing-annuity-and-growing-annuity-due/

Moyer, R. C., J.R. McGuigan, and R. Rao, *Contemporary Financial Management,* Stamford, CT, Cengage Learning, 13e, 2015

Omni Calculator (2023) accessed February 6, 2023 at: https://www.omnicalculator.com/finance/growing-annuity

Ross, Stephen A., Randolph W. Westerfield Jeffrey F. Jaffee and Bradford D. Jordon (2007) , *Corporate Finance: Core Principles and Applications,* New York, NY, McGraw-Hill Irwin, Inc. p. 115-116

Smart, S.B., W.L. Megginson, and L.J. Gitman (2007) *Corporate Finance*, 2nd Ed. Thompson Southwestern, Mason, OH, p. 72-121

Taylor, R. (1986) "Future Value of a Growing Annuity: A Note," *Journal of Financial Education,* Vol. 15 p. 17-21

BIOGRAPHIES

Terrance Jalbert, Ph.D. is Professor of Finance at University of Hawaii Hilo. He also serves as an arbitrator for the Financial Industry Regulatory Authority (FINRA). His research appears in journals including *International Journal of Finance, American Business Review, Financial Services Review, Journal of Personal Finance, Advances in Taxation, Journal of Emerging Markets, Latin American Business Review, Journal of Applied Business Research* and *The International Journal of Business and Finance Research*. He can be reached at University of Hawaii Hilo, 200 West Kawili St., Hilo, HI 96720.

Jonathan D. Stewart, Ph.D. CFA is the A. Overton Faubus Professor of Finance at Abilene Christian University. His research appears in journals including *Economic Review / Federal Reserve Bank of Atlanta, Management Accounting Quarterly, The Journal of Financial Research, The Journal of Investing, Journal of Economics and Finance Education, Advances in Financial Education, Journal of Corporate Treasury Management, International Journal of Business and Finance Research, Journal of Applied Business Research.* He can be reached at Abilene Christian University, ACU Box 29313, Abilene, TX, 79602.